Heterotic T–duality orbifolds

Stefan Groot Nibbelink

Rotterdam University of Applied Sciences

String Pheno 2022 Liverpool

talk based on

SGN, JHEP 04 (2021) 190 [arXiv:2012.02778]

SGN, Patrick Vaudrevange, JHEP 04 (2017) 030 [arXiv:1703.05323]

work under construction with Alon Faraggi, Benjamin Percival

Motivation

Motivation

21st STRING PHENOMENOLOGY Conference LIVERPOOL | 4 - 8 JULY 2022

Welcome | Registration | Program | Participants | Local Info | Getting Here

About

The 21st instalment of the String Phenomenology conference will be hosted at the Univeristy of Liverpool. This annual conference discusses recent progress in compactifications of string theory and their relation to particle physics and cosmology.

The main topics of the conference include:

- Swampland and Quantum Gravity Conjectures
- Formal and Mathematical Aspects of String Compactifications
- Heterotic Compactifications, F-theory, G2 Compactifications
- Corners of the String Landscape, e.g. Non-Geometric Compactifications
- String Model Building in Particle Physics and Cosmology
- Machine Learning Techniques in the String Landscape

Are non–geometrical compatifications really exceptions within the string landscape?

HOGESCHOOL ROTTERDAM

Motivation

Motivation for this work:

- T–duality and its heterotic extension
- Double Field Theory
- Moduli stabilisation
- T–folds and Narain or asymmetric orbifolds

Overview

Geometrical toroidal orbifolds

Elements of heterotic toroidal orbifolds

Geometry:

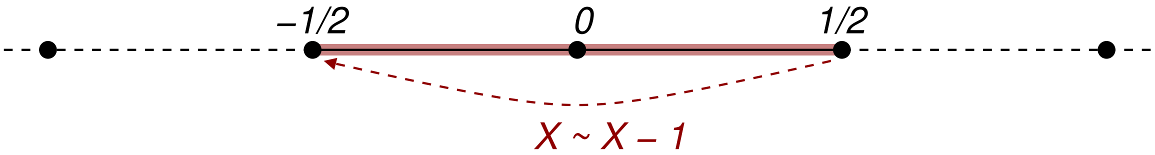
- Torus lattice
- Orbifold twist(s)
- possibly extended to roto–translation(s)

Gauge background:

- Discrete Wilson line(s)
- Gauge shift(s)

Circle S^1 compactification

A circle S^1 can defined modulo periodic identifications



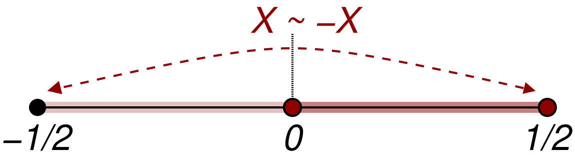
It has a metric

$$G = R^2$$

where R is its radius

Orbifold $S^1/\mathbb{Z}_2$ compactification

An orbifold $S^1/\mathbb{Z}_2$ has an additional reflectional identification



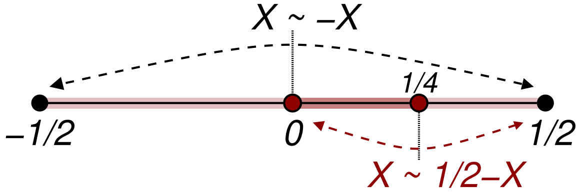
The metric

$$G = R^2$$

is unaffected by this

Orbifold $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ compactification

A second $\mathbb{Z}_2'$ may act as a roto–translation



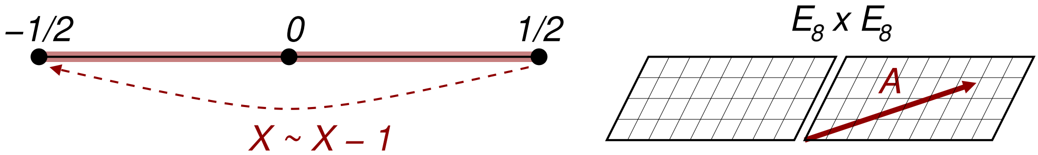
The metric

$$G = R^2$$

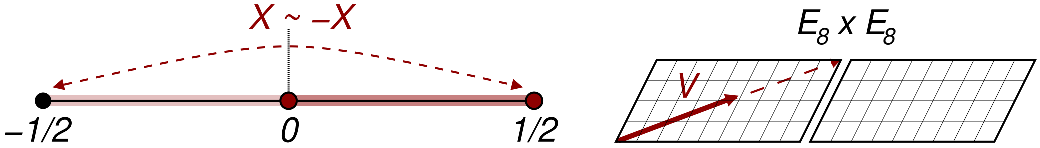
remains unaffected

Orbifold gauge backgrounds

A Wilson line is a constant $E_8 \times E_8$ gauge background associated with a translation



A gauge shift vector translation in the $E_8 \times E_8$ associated to a twist



T–duality covariant formulation

T–duality in one dimension

String theory on a circle with radius R and on a dual circle with radius $1/R$ are equivalent

Since the spectrum of Kaluza–Klein and winding modes

$$M^2 = \frac{m^2}{R^2} + n^2 R^2$$

is invariant under the simultaneous interchange:

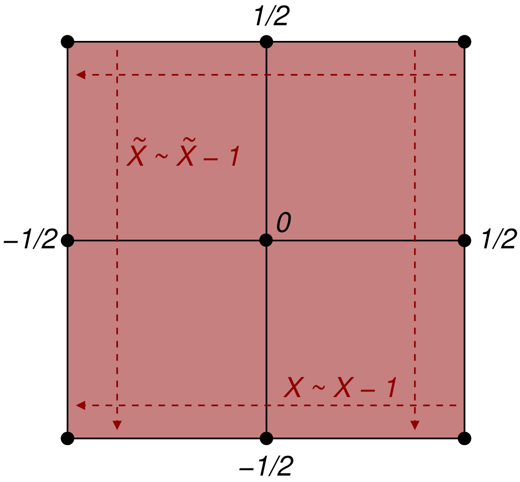
$$R \leftrightarrow \frac{1}{R}, \qquad m \leftrightarrow n$$

This is sometimes represented as replacing the coordinate with its dual:

$$X = X_L + X_R \leftrightarrow \widetilde{X} = X_L - X_R$$

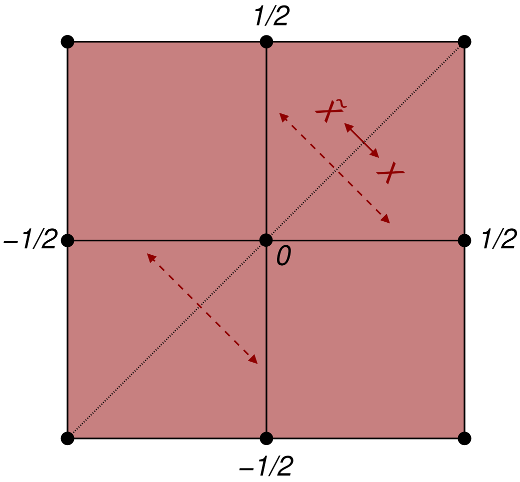
Doubled circle

Both the coordinate and its dual are subject to periodic identifications

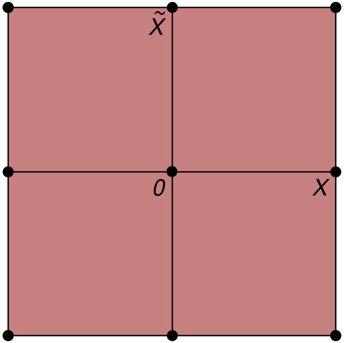


T–duality on the doubled circle

T–duality acts as a reflection in the diagonal



Doubled circle geometry



Doubled coordinates:

$$Y = \begin{pmatrix} X \\ \widetilde{X} \end{pmatrix}$$

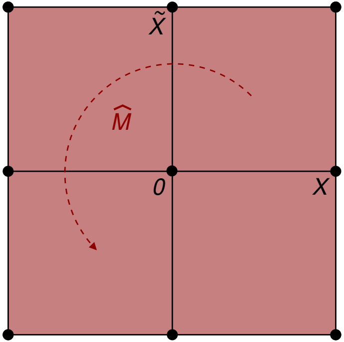
Generalised metric:

$$\widehat{\mathcal{H}} = \begin{pmatrix} R^2 & 0 \\ 0 & R^{-2} \end{pmatrix}$$

Invariant metric:

$$\widehat{\eta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

T–duality group covariant description



$O_{\widehat{\eta}}(1,1;\mathbb{Z})$ T–duality group:

$$\widehat{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,, \quad a,b,c,d \in \mathbb{Z}$$

The doubled coordinates

$$Y \to \widehat{M}^{-1}\, Y$$

and the generalised metric

$$\widehat{\mathcal{H}} \to \widehat{M}^T\, \widehat{\mathcal{H}}\, \widehat{M}$$

transform covariantly under the T–duality group, while the invariant metric is by definition inert

$$\widehat{M}^T\, \widehat{\eta}\, \widehat{M} = \widehat{\eta}$$

Double Field Theory

- Field theory using doubled coordinates:

Hull'06, Hohm,Hull,Zwiebach'10
Aldazabal,Marqués,Núñez'13

$$Y = \begin{pmatrix} X \\ \widetilde{X} \end{pmatrix} : \quad \begin{cases} \text{original coordinates} & X^T = (X^1, \ldots, X^D) \\ \text{dual coordinates} & \widetilde{X}^T = (\widetilde{X}^1, \ldots, \widetilde{X}^D) \end{cases}$$

- and the generalised and invariant metrics

$$\widehat{\mathcal{H}} = \begin{pmatrix} G - BG^{-1}B & -BG^{-1} \\ G^{-1}B & G^{-1} \end{pmatrix} \quad \text{and} \quad \widehat{\eta} = \begin{pmatrix} 0 & \mathbb{1}_D \\ \mathbb{1}_D & 0 \end{pmatrix}$$

- with manifest T–duality group, $\widehat{M} \in \mathrm{O}_{\widehat{\eta}}(D, D; \mathbb{Z})$, covariance

$$Y \mapsto \widehat{M}^{-1}\, Y\,, \qquad \widehat{\mathcal{H}} \mapsto \widehat{M}^T\, \widehat{\mathcal{H}}\, \widehat{M}\,, \qquad \widehat{M}^T\, \widehat{\eta}\, \widehat{M} = \widehat{\eta}$$

- and a “strong constraint” removing half of the doubled coordinates

Who has the right worldsheet action, Polyakov or Tseytlin?

Polyakov's worldsheet action:

Polyakov'81

$$S = \int \frac{\mathrm{d}^2\sigma}{2\pi} \Big[-\frac{1}{2}\, \eta^{\mu\nu}\, \partial_\mu X^T G\, \partial_\nu X + \frac{1}{2}\, \epsilon^{\mu\nu}\, \partial_\mu X^T B\, \partial_\nu X \Big]$$

- zero modes of $X(\sigma, \bar\sigma)$ are the target space coordinates !
- manifest Lorentz invariant !
- no trace of T–duality ?

Tseytlin's worldsheet action:

Tseytlin'90, Copland'12
Berman,Thompson'14

$$S = \int \frac{\mathrm{d}^2\sigma}{2\pi} \Big[-\frac{1}{2}\, \partial_1 Y^T\, \widehat{\mathcal{H}}\, \partial_1 Y + \frac{1}{2}\, \partial_1 Y^T\, \widehat{\eta}\, \partial_0 Y \Big]$$

- zero modes of $Y(\sigma, \bar\sigma)$ are the coordinates and their duals ?
- not manifest Lorentz invariant ?
- manifest T–duality group covariant !

Equivalence of Polyakov's and Tseytlin's worldsheet actions

- Start from Polyakov's action
- Introduce D Abelian gauge transformations — Buscher'87

$$X \mapsto X - \lambda \,, \qquad \mathcal{A}_\mu \mapsto \mathcal{A}_\mu + \partial_\mu \lambda \,, \qquad \mu = 0, 1$$

where $\lambda^T = (\lambda^1, \ldots, \lambda^D)$ are general worldsheet functions

- Choose a Lorentz non–covariant gauge to arrive at Tseytlin's action — Roček,Tseytlin'97

$$\mathcal{A}_1(\sigma_1, \sigma_0) \overset{!}{=} 0 \,, \qquad \mathcal{A}_0(0, \sigma_0) \overset{!}{=} 0$$

- This still leaves a residual zero mode gauge transformation — SGN'20

$$Y \sim Y - \Lambda_0 \,, \qquad \Lambda_0 = \widehat{M}^{-1} \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix} \,, \qquad \widehat{M} \in O_{\widehat{\eta}}(D, D; \mathbb{Z})$$

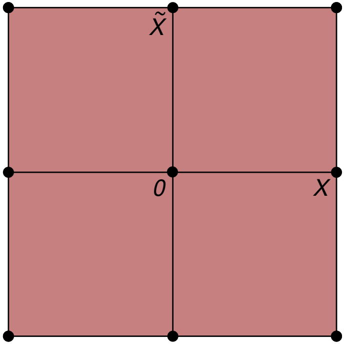
implementing a worldsheet version of the "strong constraint"

Reduction of the zero modes

The residual zero mode gauge transformation

$$Y \sim Y - \Lambda_0 \,, \quad \Lambda_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda_0 \end{pmatrix}$$

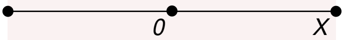
leaves the zero mode of X as the physical coordinate, since the dual coordinate $\widetilde{X}$ can be gauged away



The residual zero mode gauge transformation

$$Y \sim Y - \Lambda_0 \,, \quad \Lambda_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda_0 \end{pmatrix}$$

leaves the zero mode of X as the physical coordinate, since the dual coordinate $\widetilde{X}$ can be gauged away

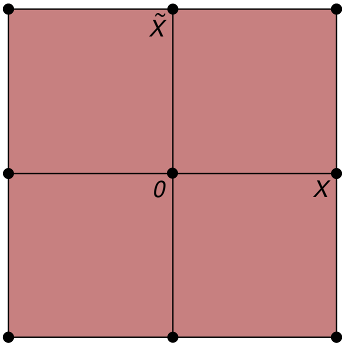


Reduction of the zero modes

The residual zero mode gauge transformation

$$Y \sim Y - \Lambda_0 \, , \quad \Lambda_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix}$$

leaves the zero mode of $\widetilde{X}$ as the physical coordinate, since the coordinate X can be gauged away

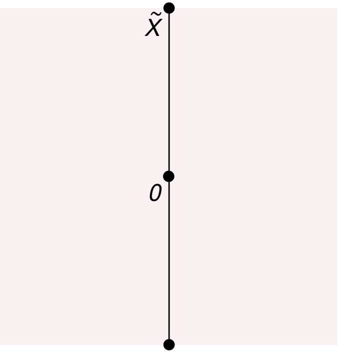


Reduction of the zero modes

The residual zero mode gauge transformation

$$Y \sim Y - \Lambda_0 \,, \quad \Lambda_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ 0 \end{pmatrix}$$

leaves the zero mode of $\widetilde{X}$ as the physical coordinate, since the coordinate X can be gauged away



Heterotic extension

- The $2D{+}16$–dimensional Narain torus, defined as $Y \sim Y + N$, $N \in \mathbb{Z}^{2D+16}$, has generalised and invariant metrics given by

Hohm,Kwak'11

SGN,Vaudrevange'17

$$\widehat{\mathcal{H}} = \begin{pmatrix} G + A^T g_{16}^{-1} A + C G^{-1} C^T & -CG^{-1} & (\mathbb{1}_D + CG^{-1}) A^T \\ -G^{-1} C^T & G^{-1} & -G^{-1} A^T \\ A(\mathbb{1}_D + G^{-1} C^T) & -AG^{-1} & g_{16} + AG^{-1} A^T \end{pmatrix} , \qquad \widehat{\eta} = \begin{pmatrix} 0 & \mathbb{1}_D & 0 \\ \mathbb{1}_D & 0 & 0 \\ 0 & 0 & g_{16} \end{pmatrix}$$

with $C{=}B{+}\frac{1}{2}A^T g_{16}^{-1} A$, Wilson lines $A = (A_i)$ and the $E_8 \times E_8$ Cartan metric g_{16}

- and manifest T–duality group, $\widehat{M} \in \mathrm{O}_{\widehat{\eta}}(D, D+16; \mathbb{Z})$, covariance

$$Y \mapsto \widehat{M}^{-1}\, Y\, , \qquad \widehat{\mathcal{H}} \mapsto \widehat{M}^T\, \widehat{\mathcal{H}}\, \widehat{M}\, , \qquad \widehat{M}^T\, \widehat{\eta}\, \widehat{M} = \widehat{\eta}$$

Distinction left– and right–movers

The distinction between left– and right–moving sectors relies on a $\mathbb{Z}_2$–grading

$$\widehat{\mathcal{Z}} = \widehat{\eta}^{-1}\widehat{\mathcal{H}}\,, \qquad \widehat{\mathcal{Z}}^2 = \mathbb{1}$$

The left–moving sector:

- Projector: $\widehat{\mathcal{P}}_\mathsf{L} = \frac{1}{2}(\mathbb{1} + \widehat{\mathcal{Z}})$
- Non–supersymmetric worldsheet
- Target space gauge degrees of freedom
- Target space matter representations

The right–moving sector:

- Projector: $\widehat{\mathcal{P}}_\mathsf{R} = \frac{1}{2}(\mathbb{1} - \widehat{\mathcal{Z}})$
- Supersymmetric worldsheet
- Target space bosons and fermions
- Target space supersymmetry

HOGESCHOOL ROTTERDAM

T–duality Narain orbifolds

A very incomplete overview of works on asymmetric orbifolds

Pioneering papers:

Narain,Sarmadi,Vafa'87,'90

Ibáñez,Mas,Nilles,Quevedo'87

Lüst,Theisen'87

Some later works:

Spaliński'92, Sasada'94, Kakushaze,Tye'96

Erler'96, Dabholkar,Harvey'98

Beye,Kobayashi,Kuwakin'13, Tan'15

Acharya,Aldazabal,Font,Narain,Zadeh'22

Asymmetric orbifolds in type–II / orientifold constructions:

Blumenhagen,Görlich'98, with,Körs,Lüst'00

Angelantonj,Blumenhagen,Gaberdiel'00, Bianchi'08

Anastasopoulos,Bianchi,Morales,Pradisi'09

Sugawara,Wada'16, Aoyama,Sugawara'20

Florakis,García-Etxebarria,Lüst,Regalado'17

Relation with double field theory / non–geometry:

Condeescu,Florakis,Lüst'12

Condeescu,Florakis,Kounnas,Lüst'13

Bakas,Lüst,Plauschinn'16

T–duality orbifold actions on Narain lattices

Point group $\theta \in \mathbf{P}$ action

$$Y \mapsto \theta[Y] = \widehat{\mathcal{R}}_\theta Y + V_\theta$$

$$\widehat{\mathcal{R}}_\theta \in O_{\widehat{\eta}}(D, D+16; \mathbb{Z})$$

preserves a generalised metric

$$\widehat{\mathcal{R}}_\theta^T \widehat{\mathcal{H}} \widehat{\mathcal{R}}_\theta = \widehat{\mathcal{H}}$$

This description combines

- Orbifold twists
- Roto–translations
- Gauge shifts

SGN,Vaudrevange'17

The representation matrices $\widehat{\mathcal{R}}_\theta$ may be block diagonalised to

$$R_\theta = \begin{pmatrix} R_{\mathrm{R}\,\theta} & 0 \\ 0 & R_{\mathrm{L}\,\theta} \end{pmatrix}$$

with $R_{\mathrm{R}\,\theta} \in \mathrm{O}(D;\mathbb{R})$ and $R_{\mathrm{L}\,\theta} \in \mathrm{O}(D+16;\mathbb{R})$ the orbifold actions on the right– and left–movers

For a symmetric orbifold this can be cast into the form

$$R_{\mathrm{L}\,\theta} = \begin{pmatrix} R_{\mathrm{R}\,\theta} & 0 \\ 0 & \mathbb{1}_{16} \end{pmatrix}$$

Counting untwisted moduli of Narain orbifolds

The number of the untwisted moduli

SGN,Vaudrevange'17

$$\dim\left[\mathfrak{M}_{\mathbf{P}}\right] = \frac{1}{|\mathbf{P}|}\sum_{\theta\in\mathbf{P}} \chi_{\mathrm{R}}(\theta)\,\chi_{\mathrm{L}}^{*}(\theta)\,, \qquad \chi_{\mathrm{L/R}}(\theta) = \mathrm{tr}\left[R_{\mathrm{L/R}\,\theta}\right]$$

is determined by the left– and right–moving characters

The maximum number is $D(D+16)$, which corresponds to

- $D(D+1)/2$ metric G components
- $D(D-1)/2$ anti–symmetric B–field components
- $16\,D$ continuous Wilson line A components

There are no free untwisted moduli when:

- The left– and right–moving characters are orthogonal

On the existence of Narain orbifolds

A Narain orbifold only exists when an orbifold compatible generalised metric can be constructed

$$\widehat{\mathcal{R}}_\theta^T\, \widehat{\mathcal{H}}(G,B,A)\, \widehat{\mathcal{R}}_\theta \overset{!}{=} \widehat{\mathcal{H}}(G,B,A)\,, \qquad \widehat{\mathcal{R}}_\theta = \begin{pmatrix} \widehat{\mathcal{R}}_{11} & \widehat{\mathcal{R}}_{12} & \widehat{\mathcal{R}}_{13} \\ \widehat{\mathcal{R}}_{21} & \widehat{\mathcal{R}}_{22} & \widehat{\mathcal{R}}_{23} \\ \widehat{\mathcal{R}}_{31} & \widehat{\mathcal{R}}_{32} & \widehat{\mathcal{R}}_{33} \end{pmatrix}$$

This lead to complicated coupled Riccati matrix equations: SGN,Vaudrevange'17

$$(G+C^T)\widehat{\mathcal{R}}_{12}(G+C^T) + A^T(\widehat{\mathcal{R}}_{32}(G+C^T) + \widehat{\mathcal{R}}_{31}) - \widehat{\mathcal{R}}_{22}(G+C^T) + (G+C^T)\widehat{\mathcal{R}}_{11} = \widehat{\mathcal{R}}_{21}$$

$$A^T\widehat{\mathcal{R}}_{32}A^T + (G+C^T)(\widehat{\mathcal{R}}_{13} + \widehat{\mathcal{R}}_{12}A^T) - \widehat{\mathcal{R}}_{22}A^T + A^T\widehat{\mathcal{R}}_{33} = \widehat{\mathcal{R}}_{23}$$

Construction of orbifold compatible generalized metrics

- Choose block diagonalised forms R_θ of the integral matrices $\widehat{\mathcal{R}}_\theta$ SGN'20
- Construct from any $(2D+16)\times(2D+16)$– matrix $\mathcal{M}_0$ an orbifold invariant one

$$\mathcal{M}_{\text{inv}} = \frac{1}{|\mathbf{P}|}\sum_{\theta\in\mathbf{P}} R_\theta^{-1}\,\mathcal{M}_0\,\widehat{\mathcal{R}}_\theta$$

- Define an invariant $\mathbb{Z}_2$–grading from it by

$$\widehat{\mathcal{Z}}_{\text{inv}} = \mathcal{M}_{\text{inv}}^{-1}\,\eta\,\mathcal{M}_{\text{inv}}~, \quad \eta = \begin{pmatrix} -\mathbb{1}_D & 0 \\ 0 & \mathbb{1}_{D+16} \end{pmatrix}~, \qquad \widehat{\mathcal{Z}}_{\text{inv}}^2 = \mathbb{1}$$

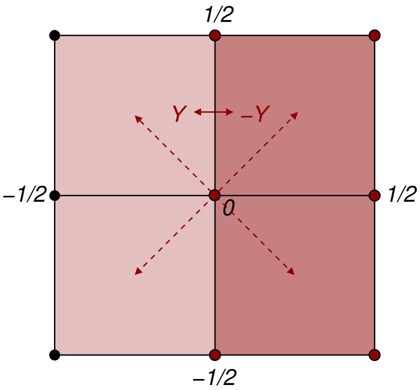
- Construct an invariant generalised metric and enforce that it is symmetric

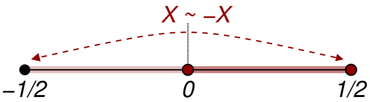
$$\widehat{\mathcal{H}}_{\text{inv}} = \widehat{\eta}\,\widehat{\mathcal{Z}}_{\text{inv}}~, \qquad \widehat{\mathcal{H}}_{\text{inv}}^T = \widehat{\mathcal{H}}_{\text{inv}}$$

Possible caveat: The obtained generalised metric might not be positive definite

Examples of T–duality Narain orbifolds

The geometric $S^1/\mathbb{Z}_2$ orbifold as a symmetric T–duality orbifold





The orbifold action $X \mapsto -X$ can be extended in doubled geometry language to

$$Y \mapsto \widehat{\mathcal{R}}\, Y \,, \quad \widehat{\mathcal{R}} = -\mathbb{1} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

This is a symmetric orbifold as

$$R_\mathrm{R} = -1 \,, \quad R_\mathrm{L} = -1$$

It looks like that in the fundamental domain of the doubled circle there are 4 fixed points

But since for example the zero modes of $\widetilde{X}$ can be gauged away only 2 are physical

Symmetric heterotic orbifold

Consider a heterotic extension of the $\mathbb{Z}_{2\text{-I}}$ orbifold in D dimensions and its associated block–diagonal representation: SGN'20

$$\widehat{\mathcal{R}} = \begin{pmatrix} -\mathbb{1}_D & 0 & 0 \\ 0 & -\mathbb{1}_D & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}, \qquad R = \begin{pmatrix} -\mathbb{1}_D & 0 & 0 \\ 0 & -\mathbb{1}_D & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}$$

- This T–duality orbifold only affects the right–movers
- Supersymmetry only when D is dividable by 4, i.e. $D = 4, 8$
- Only the Wilson line moduli are frozen:

$$\dim\left[\mathfrak{M}_{\mathbf{P}}\right] = \tfrac{1}{2}D(D+16) + \tfrac{1}{2}(-D)(-D+16) = D^2$$

HOGESCHOOL ROTTERDAM

Classification of $O_{\hat{\eta}}(1,1;\mathbb{Z})$ point groups

$O_{\hat{\eta}}(1,1;\mathbb{Z})$ T–duality elements

$$\hat{\mathcal{R}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad a,b,c,d \in \mathbb{Z}$$

satisfy $\hat{\mathcal{R}}^T \hat{\eta} \hat{\mathcal{R}} = \hat{\eta}$, concretely

$$ac = bd = 0\,, \qquad ad + bc = 1$$

There are only 4 choices:

$$\hat{\mathcal{R}} = \pm\mathbb{1}\,, \quad \pm\hat{I}\,,$$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Consequently, the possible point groups are

Group	$\mathbb{Z}_{2\text{-I}}$	$\mathbb{Z}_{2\text{-II}}$	$\mathbb{Z}_{2\text{-III}}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
Elements	$\{\mathbb{1}, -\mathbb{1}\}$	$\{\mathbb{1}, \hat{I}\}$	$\{\mathbb{1}, -\hat{I}\}$	$\{\pm\mathbb{1}, \pm\hat{I}\}$
(A)Symmetric	S	A	A	A
Generator(s)	$-\mathbb{1}$	$\hat{I}$	$-\hat{I}$	$-\mathbb{1}$ $\hat{I}$
R_{R}	-1	-1	$+1$	-1 -1
R_{L}	-1	$+1$	-1	-1 $+1$

The symmetric $\mathbb{Z}_{2\text{-I}}$ orbifold was considered in the previous slide

Classification of $O_{\hat{\eta}}(1,1;\mathbb{Z})$ point groups

Some comments:

- There is only 1 symmetric but 3 asymmetric orbifolds
- The distinction between $\mathbb{Z}_{2\text{–II}}$ and $\mathbb{Z}_{2\text{–III}}$ is significant in heterotic context
- The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold has a symmetric and an asymmetric generator

Consequently, the possible point groups are

Group	$\mathbb{Z}_{2\text{–I}}$	$\mathbb{Z}_{2\text{–II}}$	$\mathbb{Z}_{2\text{–III}}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
Elements	$\{\mathbb{1}, -\mathbb{1}\}$	$\{\mathbb{1}, \widehat{I}\}$	$\{\mathbb{1}, -\widehat{I}\}$	$\{\pm\mathbb{1}, \pm\widehat{I}\}$
(A)Symmetric	S	A	A	A
Generator(s)	$-\mathbb{1}$	$\widehat{I}$	$-\widehat{I}$	$-\mathbb{1}$ $\widehat{I}$
R_R	-1	-1	$+1$	-1 -1
R_L	-1	$+1$	-1	-1 $+1$

Classification of $O_{\hat{\eta}}(1,1;\mathbb{Z})$ point groups

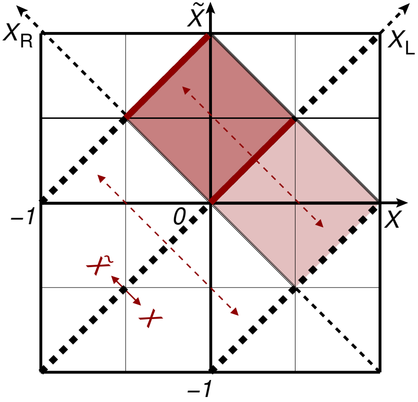
Some comments:

- There is only 1 symmetric but 3 asymmetric orbifolds
- The distinction between $\mathbb{Z}_{2\text{–II}}$ and $\mathbb{Z}_{2\text{–III}}$ is significant in heterotic context
- The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold has two asymmetric generators

Consequently, the possible point groups are

Group	$\mathbb{Z}_{2\text{–I}}$	$\mathbb{Z}_{2\text{–II}}$	$\mathbb{Z}_{2\text{–III}}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
Elements	$\{\mathbb{1}, -\mathbb{1}\}$	$\{\mathbb{1}, \widehat{I}\}$	$\{\mathbb{1}, -\widehat{I}\}$	$\{\pm\mathbb{1}, \pm\widehat{I}\}$
(A)Symmetric	S	A	A	A
Generator(s)	$-\mathbb{1}$	$\widehat{I}$	$-\widehat{I}$	$-\widehat{I}$ $\quad \widehat{I}$
R_{R}	-1	-1	$+1$	$+1$ $\quad -1$
R_{L}	-1	$+1$	-1	-1 $\quad +1$

Right–twisted T–duality orbifold



The $\mathbb{Z}_{2\text{–II}}$ orbifold action on the doubled coordinates reads

$$Y \mapsto \widehat{\mathcal{R}}\, Y \,, \quad \widehat{\mathcal{R}} = \widehat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

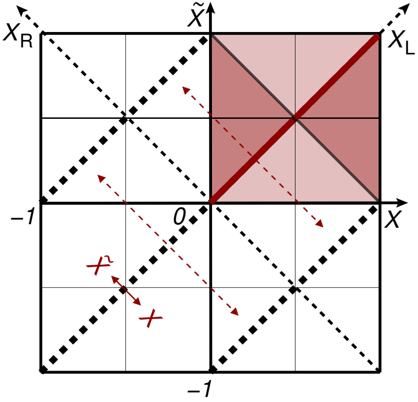
This is an asymmetric orbifold as

$$R_R = -1 \,, \quad R_L = +1$$

It only acts in the direction X_R, hence there seems to be two fixed lines

But leads just to a single twisted sector as they are connected by a lattice shift

Right–twisted T–duality orbifold



The $\mathbb{Z}_{2\text{–II}}$ orbifold action on the doubled coordinates reads

$$Y \mapsto \widehat{\mathcal{R}}\, Y\,, \quad \widehat{\mathcal{R}} = \widehat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

This is an asymmetric orbifold as

$$R_R = -1\,, \quad R_L = +1$$

The fundamental domain takes the form of a möbius strip in the doubled geometry

This is a T–fold: there is no global projection on only the coordinate or its dual

Right–twisted full T–duality Narain orbifold

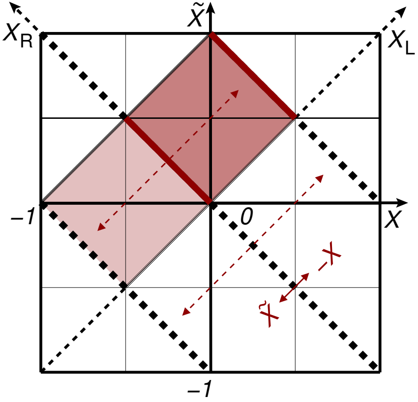
Consider a heteronic extension of the $\mathbb{Z}_{2\text{-II}}$ orbifold twist in D dimensions and its associated block–diagonal representation: SGN'20

$$\widehat{\mathcal{R}} = \begin{pmatrix} 0 & \mathbb{1}_D & 0 \\ \mathbb{1}_D & 0 & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}, \qquad R = \begin{pmatrix} -\mathbb{1}_D & 0 & 0 \\ 0 & \mathbb{1}_D & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}$$

- This T–duality orbifold only affects the right–movers
- Supersymmetric in target space only when D is dividable by 4, i.e. $D = 4, 8$
- All untwisted moduli are frozen:

$$\dim\left[\mathfrak{M}_{\mathbf{P}}\right] = \tfrac{1}{2}D(D+16) + \tfrac{1}{2}(-D)(D+16) = 0$$

Left–twisted T–duality orbifold



The $\mathbb{Z}_{2\text{–III}}$ orbifold action on the doubled coordinates reads

$$Y \mapsto \widehat{\mathcal{R}}\, Y\,, \quad \widehat{\mathcal{R}} = -\widehat{I} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

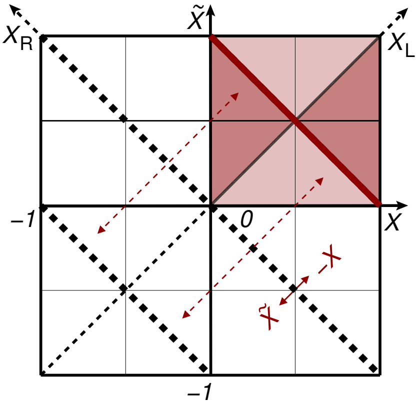
This is an asymmetric orbifold as

$$R_\mathrm{R} = +1\,, \quad R_\mathrm{L} = -1$$

It only acts in the direction X_L, hence there seems to be two fixed lines

But leads just to a single twisted sector as they are connected by a lattice shift

Left–twisted T–duality orbifold



The $\mathbb{Z}_{2\text{-III}}$ orbifold action on the doubled coordinates reads

$$Y \mapsto \widehat{\mathcal{R}}\, Y\,, \quad \widehat{\mathcal{R}} = -\widehat{I} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

This is an asymmetric orbifold as

$$R_{\mathrm{R}} = +1\,, \quad R_{\mathrm{L}} = -1$$

The fundamental domain takes the form of a möbius strip in the doubled geometry

This is a T–fold: there is no global projection on only the coordinate or its dual

Left–twisted full T–duality Narain orbifold

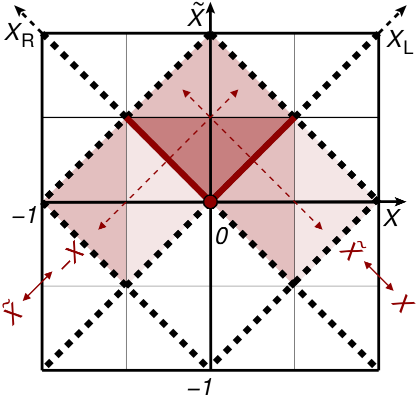
Consider a heteronic extension of the $\mathbb{Z}_{2\text{-III}}$ orbifold twist in D dimensions and its associated block–diagonal representation: SGN'20

$$\widehat{\mathcal{R}} = \begin{pmatrix} 0 & -\mathbb{1}_D & 0 \\ -\mathbb{1}_D & 0 & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}, \qquad R = \begin{pmatrix} \mathbb{1}_D & 0 & 0 \\ 0 & -\mathbb{1}_D & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}$$

- No action on the right–moving and the gauge degrees of freedom
- All target space supersymmetries are preserved for any D
- Only the Wilson line moduli are unfixed:

$$\dim\left[\mathfrak{M}_{\mathbf{P}}\right] = \tfrac{1}{2}D(D+16) + \tfrac{1}{2}D(-D+16) = 16\,D$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ left– / right–twisted T–duality orbifold



The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold action on the doubled co-ordinates reads

$$Y \mapsto \widehat{I}\, Y\,, \quad Y \mapsto -\widehat{I}\, Y\,, \quad \widehat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

both acting asymmetrically

Their fixed lines correspond to a single twisted sector each

$\mathbb{Z}_2 \times \mathbb{Z}_2$ left– / right–twisted T–duality orbifold

Consider a heterotic extension of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists in D dimensions and its associated block–diagonal representation:

$$\widehat{\mathcal{R}}_{\widehat{I}} = \begin{pmatrix} 0 & \mathbb{1}_D & 0 \\ \mathbb{1}_D & 0 & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}, \quad \widehat{\mathcal{R}}_{-\widehat{I}} = \begin{pmatrix} 0 & -\mathbb{1}_D & 0 \\ -\mathbb{1}_D & 0 & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}; \qquad R_{\widehat{I}} = \begin{pmatrix} -\mathbb{1}_D & 0 & 0 \\ 0 & \mathbb{1}_D & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}, \quad R_{-\widehat{I}} = \begin{pmatrix} \mathbb{1}_D & 0 & 0 \\ 0 & -\mathbb{1}_D & 0 \\ 0 & 0 & \mathbb{1}_{16} \end{pmatrix}$$

- This T–duality orbifold affects both the left– and right–movers
- Supersymmetric in target space only when D is dividable by 4, i.e. $D = 4, 8$
- All untwisted moduli are frozen:

$$\dim \left[\mathfrak{M}_{\mathbf{P}}\right] = \tfrac{1}{4}D(D+16) + \tfrac{1}{4}(-D)(D+16) + \tfrac{1}{4}D(-D+16) + \tfrac{1}{4}(-D)(-D+16) = 0$$

Bound on the generator order

If the order K of an orbifold twist satisfies Vaidyanathaswamy'1928

$$\phi(K) \leq D_\Gamma \ ,$$

where $\Phi(K)$ is the Euler ϕ–function, then there exists at least one lattice Γ of dimension D_Γ with rotational symmetry of that order

Geometric orbifold				
D	1	2	4	6
K	2	6	12	18

Narain orbifold				
$2D$	2	4	8	12
K	6	12	30	42

Hence, the order of the generators of asymmetric orbifolds might be significantly higher than of the ordinary orbifolds SGN,Vaudrevange'17

For example, the largest Abelian point group of a 2D symmetric orbifold is $\mathbb{Z}_6$, while the asymmetric variant is $\mathbb{Z}_{12}$

Towards a classification of heterotic T–duality orbifolds

Classification of symmetric orbifolds up to dimension 6

Opgenorth,Plesken,Schulz'98

- $\mathbb{Q},\mathbb{Z}$– and affine–classes
- Lattices, twists and roto–translations
- Supersymmetric subset known — Fischer,Ratz,Torrado,Vaudrevange'12
- i.e. a purely geometrical classification

A classification of asymmetric orbifolds could be performed along similar lines

SGN,Vaudrevange'17

- Narain $\mathbb{Q},\mathbb{Z}$–classes and Poincaré classes
- Narain lattices, twists, roto–translations
- but these now also include (discrete) Wilson lines and gauge shift embeddings
- i.e. a classification of all heterotic asymmetric orbifold models

Conclusions

Summary

Heterotic T–duality orbifolds provide a view on non–geometric backgrounds

- They give an heterotic extension of double field theory constructions
- The Tseytlin worldsheet action provides a worldsheet starting with a residual zero mode gauge symmetry to implement the "strong constraint"

Their existence relies on the existence of an invariant generalised metric

- A procedure to obtain this metric makes use of block diagonalised orbifold twists

Classification ingredients ($\mathbb{Q}, \mathbb{Z}$– and Poincaré classes) for them were outlined

The given T–duality orbfiold examples illustrate e.g. that

- some or all untwisted moduli can be frozen
- gauge symmetry and supersymmetry breaking can be decoupled
- their number is presumably (much) larger than symmetric orbifolds

Outlook

Currently under investigation:

- Classification of $\mathbb{Z}_2^N$ asymmetric orbifolds — with Percival
- Matching with free fermionic models — with Faraggi

  like for symmetric orbifolds — Athanasopoulos,Faraggi,SGN,Mehta'16

Further applications:

- String origin flavour symmetry — Baur,Nilles,Trautner,Vaudrevange'19
- Orbifolds that exchange E_8 factors — Font,Fraiman,Graña,Núñez,Freitas'21
- ....

Thank you for your attention!

Questions?